90636

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2004

90636 Integrate functions and solve problems by integration, differential equations or numerical methods

Credits: Six
9.30 am Tuesday 23 November 2004

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.
Show the results of any integration needed to solve the problems.

If you need more space for any answer, use the pages provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement Criteria	For Assessor's use only	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Integrate functions and solve problems by integration, differential equations or numerical methods.	Find integrals and use integration to solve problems.	Use a variety of integration techniques to solve problem(s).
Overall Level of Performance		

You are advised to spend 50 minutes answering the questions in this booklet.

Show **ALL** working.

QUESTION ONE

Find the integrals. You do not need to simplify your answers.

Do not forget any arbitrary constants.

(a) $\int (\frac{4}{x^2} + \frac{3}{x}) \, dx$

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(b) $\int (3t - 1)^5 \, dt$

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(c) $\int \sec^2 4x \, dx$
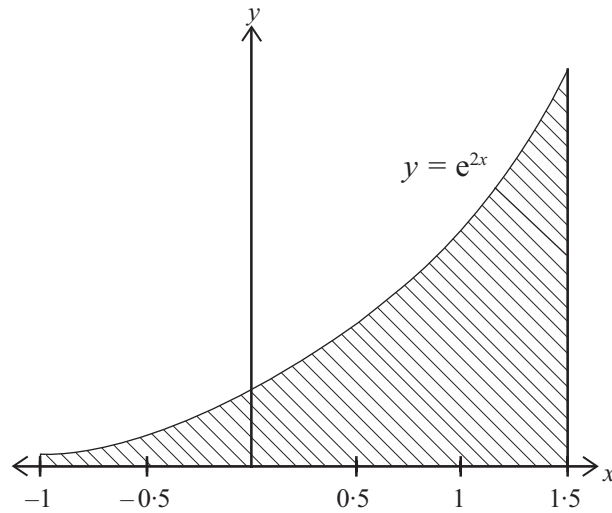
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QUESTION TWO

Part of the school's flower garden can be modelled by the shaded area on the graph.
The shaded area is bounded by $y = e^{2x}$, $x = -1$ and $x = 1.5$.

The gardener wants to fertilise the garden.

Calculate the area of the flower garden that he wants to fertilise, if x and y are measured in metres.

QUESTION THREE

The graph shows the velocity, $v(t)$, of a cyclist over some time.

Using Simpson's Rule, find an approximation for the distance, $s(t)$, travelled by the cyclist,

where $s(t) = \int_1^3 v(t)\,dt$.

Velocity of a cyclist

$v(t)$
$(km\ h^{-1})$

t (hours)

QUESTION FOUR

An object is oscillating in a straight line through the origin.

Its velocity is given by $\dfrac{ds}{dt} = \sin\left(\dfrac{t}{2} - \dfrac{1}{2}\right)$

where t is the time in seconds

and s is the distance in metres from the origin.

The object passes through the origin one second from the start.

Find an equation for the distance of the object from the origin at any time t.

QUESTION FIVE

Find $\int x\sqrt{1+x}\ dx$. A suitable substitution may be helpful.

QUESTION SIX

The shape of a musical instrument can be found by rotating the curve $y = x^3 + 4$ through $360°$ about the y-axis between $y = 4$ and $y = h$, where $h > 4$.

Find an expression for the volume of this shape in terms of h.

QUESTION SEVEN

Find the value of k where $x = k$ is a vertical line that divides the area between $y = \sqrt{x}$,

$x = 9$ and the x-axis into two equal parts.

QUESTION EIGHT

In Newtown the population of the town, N, changes at a rate proportional to the population of the town.

The population of the town in 1974 was 50 000 people and in 2004 it is 90 000 people.

Find an equation for the population, N, in terms of time t, where t is the number of years after 1974.

QUESTION NINE

A solid plastic cylinder, radius a centimetres ($a > 2$) and length 4π centimetres, is centred on the line $y = a$.

The volume of the plastic cylinder has been reduced by rotating the function $y = \sin\dfrac{1}{4}x$ through 360° about the line $y = a$ and removing all the material outside this shape.

The plastic then has a hole, radius 0.5 cm, drilled along $y = a$, and this material is removed.

Find the volume of the remaining plastic.

**Extra paper for continuation of answers if required.
Clearly number the question.**

Question number

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Clearly number the question.**

Question
number